New Methods for Predictability Analysis

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LONG-TERM GOAL

Our long-term goal is to improve the accuracy of atmospheric and oceanic deterministic and statistical forecast. Specifically, we seek to maximize deterministic forecast accuracy while minimizing observational and computational cost. We also seek to maximize the accuracy and utility of statistical forecasts, which can provide forecast information beyond the time interval over which deterministic forecast is possible.

OBJECTIVES

Our objective is to use fundamental advances in understanding of deterministic and statistical forecast dynamics to develop practical methods for improving forecast accuracy. In particular we seek to exploit model order reduction techiques derived from control theory together with the formal equivalence between 4D-Var and the extended Kalman filter to implement an approximate optimal state estimation method for forecast initialization. We seek to improve the accuracy of ensemble forecasts by accounting for model uncertainty in the choice of ensembles. We are developing methods to improve the accuracy of forecast of atmospheric statistics including the mean strength and preferred location of cyclones and their associated heat and momentum transports by identifying the sensitivity of these quantities to changes in boundary conditions such as sea surface temperature. We seek to improve forecast accuracy by quantifying the effect of model error when the rror is distributed over the forecast interval.

APPROACH

We employ both analytical and numerical methods, primarily derived from the theory of dynamical systems, the theory of stochastic differential equations, and control theory to study error growth in deterministic forecast. In addition we use methods drawn from statistical analysis of certain and uncertain systems to address issues related to the statistical predictability of weather and climate. Our approach is first to improve fundamental understanding and then use this theoretical base to develop practical implementations of advanced forecast methods such as optimal state estimation algorithms. One approach we are pursuing is to reduce the dimension of the error system so that an approximate Kalman filter can be implemented. With the objective of practical application we are particularly interested in developing algorithms for optimal state estimation that utilize forecast products and

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algorithms such as the adjoint integrator that is already available at many forecast centers for use in operational variational data assimilation.

The foundation of the approach we use is provided by recent theoretical advances in non-normal time-dependent stability analysis. We use these advances together with methods of modern control theory, specifically balanced Hankel operator truncation methods, to reduce the dimension of the error system. Having reduced the order of the error system we then use this reduced order system to obtain an effective Kalman gain for state identification in the full forecast system. Implementing these methods in operational forecast environments requires modifying theoretically optimal methods to use available resources to best advantage.

We are developing theory and methods for incorporating the effects of uncertainty in the forecast error system. Uncertainty can arise in the forecast system both from incomplete knowledge of the forecast trajectory and from parameterized physics. Ensemble forecast is improved by accounting for model uncertainty in forecast error growth which differs from the more familiar uncertainties associated with initial conditions. We apply recent theoretical advances in non-normal time-dependent stability analysis to the problem of error growth in uncertain forecast systems. Optimal excitation theory is then used to construct optimal ensembles.

We are developing theory for determining the sensitivity of atmospheric statistics to structured changes such as variations in mean jet strength or in dissipation. The method used builds on previous work in which storm track statistics were obtained using a stochastic model of jet turbulence. These results extend generalized stability theory to address statistical stability of jets.

We are developing a theory for the growth of forecast error produced by distributed forcing over the forecast interval such as is produced by model error.

We are developing a theory for the growth of assimilation system error using the equivalent stabilized observer system.

The above-described work is a joint effort between Professors Brian Farrell and Petros Ioannou.

WORK COMPLETED

We have completed work on the fundamental theory for error growth in time dependent systems.

We have developed a theory of jet stream variations and how jets transition from predictable to unpredictable.

We have completed the theory for optimal reduction of the error system order based on balanced truncation of the error dynamics and have shown that error dynamics in these reduced equations accurately models time dependent forecast system error.

We have adapted optimal balanced Hankel operator model order reduction previously used in discrete time independent engineering systems to the time dependent forecast error dynamical system.

We have constructed a reduced order Kalman filter using balanced truncation, demonstrated its ability to accurately estimate the state of the model forecast, and obtained an implementation using operational forecast algorithms.

We have developed the theory of error growth in uncertain forecast systems, obtained the error growth and structure under the influence of statistically distributed parameterizations, and obtained the statistically optimal structure for use in ensemble generation.

We have completed the theory of sensitivity of forecast statistics to structured changes of the system operator and used this result to characterize statistical jet stream variance changes.

We have completed the theory of distributed forcing of error systems obtaining the deterministic and stochastic forcing that results in maximum error growth. This theory is necessary for characterizing the model error contribution to overall forecast error.

RESULTS

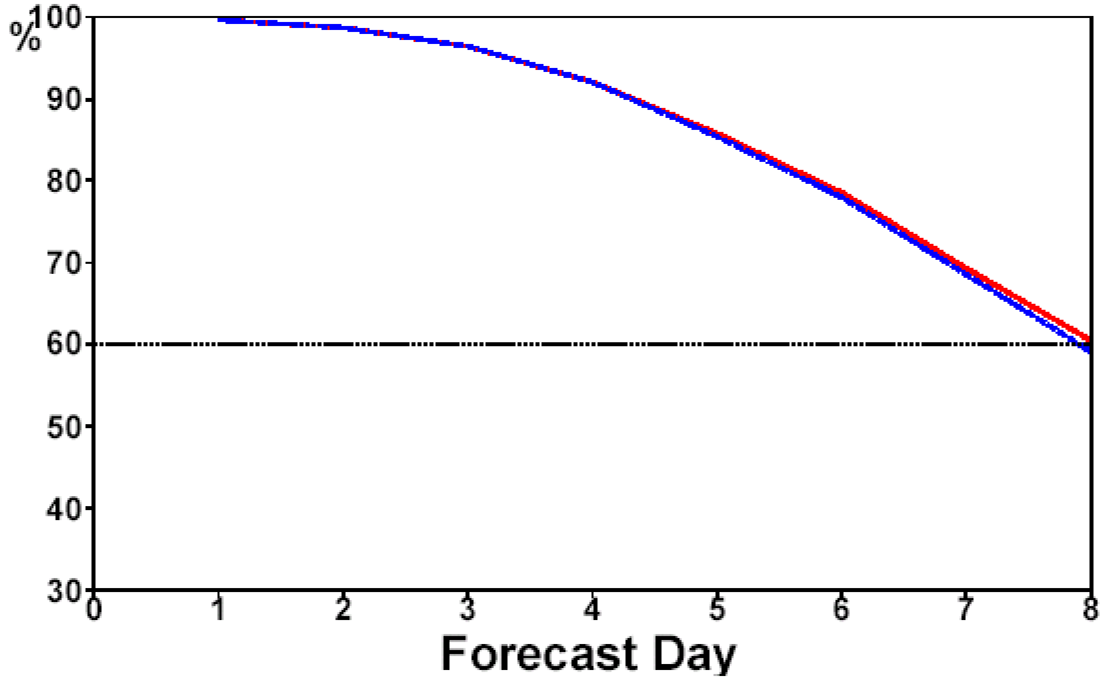
We have developed a stability theory for time dependent systems that allows better prediction of forecast error growth. We have shown that the dominant error growth arises from destabilization of a restricted set of non-normal vectors of the mean operator by time dependence (Farrell and Ioannou, 1999). These results allow us to reduce the dimension of the unstable dynamics of time dependent tangent linear error systems by representing the dynamics in a restricted subspace.

We have developed a method for reducing the dimension of the time independent error system based on retaining the dominant error subspace. This method truncates the error growth dynamics in balanced coordinates (Farrell and Ioannou, 2001a).

We have implemented and tested the balanced truncation method in model problems. We have found in our model problems that a very good approximation to the error dynamics is obtained with substantial dimension reduction (Farrell and Ioannou, 2001a). Preliminary work at ECMWF suggests that propagation of error covariance of the forecast system using this method can be accurately achieved by retaining $O(10^3)$ degrees of freedom.

We have determined that truncating the dynamical system in a balanced realization of the optimals and evolved optimals for a single appropriately chosen time provides a nearly optimal reduced order error system. This is a significant result because it suggests that implementing the order reduction algorithm in an operational forecast mode can be greatly simplified (Farrell and Ioannou, 2001a).

We have extended the balanced truncation method of optimal order reduction to a time dependent Lyapunov unstable error system (Farrell and Ioannou, 2001b).



This figure compares the assimilation obtained using 4D-Var at the full ECMWF operational resolution with the assimilation obtained using an order 100 rank reduced order Kalman filter using balanced truncation. Shown are the associated anomaly correlation scores for 500hPa geopotential in the northern hemisphere as a function of forecast day. The upper curve (red) corresponds to the reduced order Kalman filter which reaches a 60% anomaly correlation by day 8 of the forecast, while the lower curve (blue) shows the performance of the 12-hour 4D-Var which reaches a 60% anomaly correlation by day 7.8. The impact of the reduced order Kalman filter on the forecast score is positive despite the fact that in this experiment only 100 vectors were retained in the error covariance evolution. We estimate that a good representation of the covariance evolution can be achieved by retaining O(1000) vectors (The graph is courtesy of Michael Fisher).

We have obtained a reduced order Kalman filter in a model time dependent tangent linear forecast system. We showed that this reduced order Kalman filter successfully observes a model of the atmospheric error state (Farrell and Ioannou, 2001b).

We have in cooperation with ECMWF completed preliminary implementation of this reduced order Kalman filter in the ECMWF forecast model with encouraging preliminary results (Fisher et al, 2003).

We have developed the theory of error dynamics in uncertain systems for application to the ensemble forecast problem and obtained exact dynamical equations for the evolution of an ensemble mean field and the evolution of the ensemble covariance under uncertain dynamics (Farrell and Ioannou, 2002a and 2002b).

We have solved the problem of optimal excitation of uncertain systems. We first proved that the optimal excitation problem for uncertain systems has a solution: in uncertain systems there is a sure initial condition producing the greatest expected perturbation growth and a sure structure that is most effective in exciting variance when this structure is continuously forced. We then obtained a practical algorithm for finding these statistical optimals (Farrell and Ioannou, 2002c). These results are being used to develop a method for optimal ensemble generation.

We have completed the theory of statistical sensitivity of jets and obtained the method for calculating sensitivity of statistics to structured operator changes (Farrell and Ioannou, 2003a;b).

We have completed the theory of distributed forcing of error systems obtaining the deterministic and stochastic forcing that results in maximum error growth (Farrell and Ioannou, 2003c).

IMPACT/APPLICATION

Our results are presently being used to implement advanced methods of state estimation in forecast models including approximations to the Kalman filter. Our methods for accounting for model uncertainty and distributed forcing are directly applicable to improving ensemble forecast methods currently being used. Our results on sensitivity of operators to structured changes and on jet stability provide a new foundation for understanding the influences on as well as for making forecasts of statistical quantities.

TRANSITIONS

Our method for obtaining more accurate initial conditions has been configured to use operational forecast products and is presently being implemented in an operational forecast at ECMWF.

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